A Hadamard matrix of order 4792

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Abstract

We construct a Hadamard matrix of order $4792 = 8 \cdot 599$, an order for which no Hadamard matrix was previously known. The order is twice 2396, the smallest order for which no Hadamard matrix is known, and it was one of the last two open orders of the form $8p$ below $10\,000$. The matrix is a Goethals–Seidel array with a border, built from four ± 1 functions on the abelian group $\mathbb{Z}_7 \times \mathbb{Z}_{19} \times \mathbb{F}_9$ of order 1197 whose periodic autocorrelations sum to -4 at every nonzero shift. The four functions are invariant under a group of multipliers of order 36, so they are given by twelve 17-bit masks. We verify the autocorrelation identity by counting differences, and we check that $HH^{\mathsf{T}} = 4792I$ with three independent programs. The functions were found by a computer search in a family of symmetric structures proposed by Kumar, who used the same construction to give a Hadamard matrix of order $5912 = 8 \cdot 739$.

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Keywords. Hadamard matrix, Goethals–Seidel array, supplementary difference sets, periodic autocorrelation, computer search

AI use and author contributions

The construction and the search and verification programs in this paper were developed and run by Astra (GPT-6.2 Astra, OpenAI) from Charles O'Neill's instructions, starting from Kishan Kumar's public search programs. The manuscript was prepared from the project's research records with Claude (Anthropic). All computations were re-run before release, and the programs, the certificate and the matrix are available at https://github.com/charlieoneill11/maths-releases.

Charles O'Neill chose the problem and directed the search. Charles O'Neill has checked the arguments and takes full responsibility for the correctness of the results.

1 Introduction

A *Hadamard matrix* of order n is an $n \times n$ matrix H with entries ± 1 and $HH^{\mathsf{T}} = nI$. Such a matrix can exist only for $n = 1$, $n = 2$ and $n \equiv 0 \pmod 4$ [4], and the Hadamard conjecture, first stated by Paley [7], asserts that it exists for every such n. Sylvester's doubling [9] turns a Hadamard matrix of order n into one of order $2n$, Paley's constructions [7] cover orders $q+1$ and $2(q+1)$ for suitable prime powers q, and a long list of further constructions, many of them computer searches over arrays of commuting blocks, covers most other small orders; see [8, 1]. In 2005 Kharaghani and Tayfeh-Rezaie found a Hadamard matrix of order 428 [5], and 668 remained the smallest open order for about twenty years. In 2026 many of the smallest open orders were settled; the survey of Egan, Kotsireas, London and Ó Catháin [1] and Kumar's tally [6] record them.

As of 25 November 2026, the smallest order for which no Hadamard matrix is known is $2396 = 4 \cdot 599$ [2, 6]. Twice that order was open as well: the Hadamard Matrix Explorer lists 4792 as unknown [2], and Kumar's search records [6] list $4792 = 8 \cdot 599$ and $5752 = 8 \cdot 719$ as the two remaining open orders of the form $8p$ below $10\,000$, after his construction of order $5912 = 8 \cdot 739$. We settle this order.

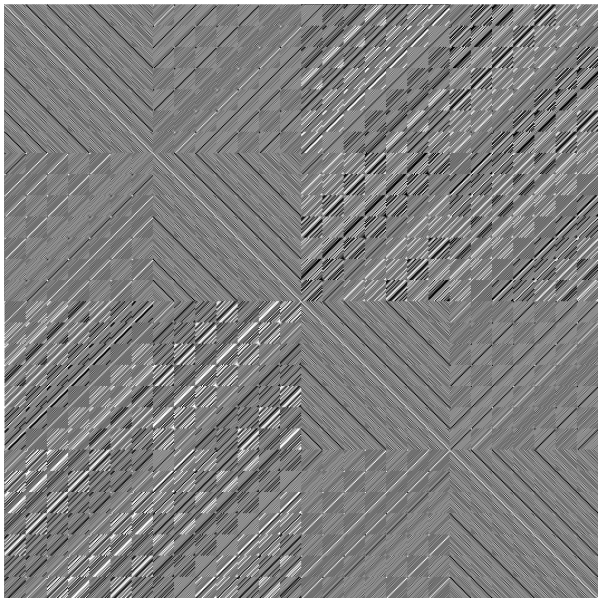
Theorem 1. *There is a Hadamard matrix of order* 4792.

A Hadamard matrix of order 2396 would give one of order 4792 by doubling. Our matrix does not arise in this way, and order 2396 remains open. Figure 1 shows the matrix.

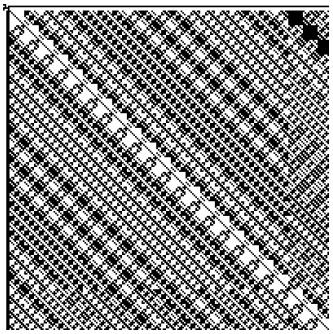
The construction. Let G be a finite abelian group of odd order m. Suppose that four functions $a, b, c, d \colon G \to \{\pm 1\}$ have periodic autocorrelations summing to -4 at every nonzero shift. Then a Goethals–Seidel array of the four group-developed matrices, with a border of width four, is a Hadamard matrix of order $4(m+1)$ (Theorem 3). Kumar calls this Spence's construction, and used it for his matrix of order 5912 with $m = 1477$ [6]. Bordered Goethals–Seidel arrays of this kind also appear in several of the constructions surveyed in [1]. Since the proof is short we give it in full. For order 4792 we take $m = 1197$ and

$$G = \mathbb{Z}_7 \times \mathbb{Z}_{19} \times \mathbb{F}_9.$$

Section 3 gives four functions on G with the required autocorrelations. They are invariant under a group of multipliers of order 36, which has 51 orbits on



(a) The whole matrix



(b) The top-left 200×200 corner

Figure 1: The Hadamard matrix H of order 4792, with $+1$ white and -1 black, in the row and column order of Theorem 3: the four border rows and columns come first, followed by the 4×4 array of blocks of size 1197. The image is drawn from the published matrix (Section 4) and agrees with it entry by entry.

G, so each function is described by 51 signs. Kumar's search notes identify this group G and this multiplier group as the most promising structure for $p = 599$ [6].

Verification. The proof of Theorem 1 is Theorem 3 together with a finite computation (Lemma 4): for each of the 1196 nonzero elements t of G, count the ordered pairs in the four negative sets that differ by t. This takes a fraction of a second. We also checked the matrix itself. Three independent programs, one of them Kumar's, build or read all 4792^2 entries and find $HH^{\mathsf{T}} = 4792I$ exactly (Section 4). The matrix, the certificate and the programs are available at `https://github.com/charlieoneill11/maths-releases`.

The search. Section 5 describes how the functions were found. The search split the problem into 1141 cases, discarded 617 of them by exact tests on two quotients of G, and found the solution in one of the remaining cases. A second case gave a second solution. The correctness of Theorem 1 does not depend on the search.

Notation. Groups are written additively. For a function $x\colon G \to \{\pm 1\}$ we write $s_x = \sum_{g\in G} x(g)$ for its sign sum,

$$P_x(t) = \sum_{g\in G} x(g)\, x(g+t) \qquad (t \in G)$$

for its periodic autocorrelation, and $[x]$ for the $G \times G$ matrix with entries $[x]_{g,h} = x(h-g)$. We write I and J for the identity and all-ones matrices, $\mathbf{1}$ for the all-ones column vector, and R for the $G \times G$ permutation matrix with $R_{g,h} = 1$ exactly when $h = -g$. Matrices indexed by G use a fixed ordering of G.

2 The bordered array

Lemma 2. *Let G be a finite abelian group and let $x, y\colon G \to \{\pm 1\}$. Then*

1. *$[x][y] = [y][x]$ and $[x]^{\mathsf{T}}[y] = [y][x]^{\mathsf{T}}$;*
2. *$[x][x]^{\mathsf{T}} = [x]^{\mathsf{T}}[x]$, and its (g,h) entry is $P_x(g-h)$;*
3. *$R^2 = I$, $R^{\mathsf{T}} = R$, $R[x]R = [x]^{\mathsf{T}}$, and $[x]R$ is symmetric;*
4. *$[x]\mathbf{1} = [x]^{\mathsf{T}}\mathbf{1} = s_x\mathbf{1}$.*

Proof. For a function $f\colon G \to \mathbb{Z}$ let L_f be the matrix with entries $(L_f)_{g,h} = f(h-g)$. Then $(L_f L_{f'})_{g,h} = \sum_k f(k-g)f'(h-k) = \sum_u f(u)f'(h-g-u)$, which is symmetric in f and f' because G is abelian, so all the matrices L_f commute. Since $[x] = L_x$ and $[x]^{\mathsf{T}} = L_{\tilde{x}}$ with $\tilde{x}(g) = x(-g)$, this gives 1 and the first part of 2. Next, $([x][x]^{\mathsf{T}})_{g,h} = \sum_k x(k-g)x(k-h) = \sum_u x(u)x(u+g-h) = P_x(g-h)$. For 3, R is the matrix of the involution $g \mapsto -g$, and $(R[x]R)_{g,h} = [x]_{-g,-h} = x(g-h) = [x]^{\mathsf{T}}_{g,h}$. Hence $([x]R)^{\mathsf{T}} = R[x]^{\mathsf{T}} = R(R[x]R) = [x]R$. Finally every row and every column of $[x]$ contains each value $x(u)$ exactly once. □

Theorem 3. *Let G be a finite abelian group of odd order m, and let $a, b, c, d\colon G \to \{\pm 1\}$ satisfy*

$$P_a(t) + P_b(t) + P_c(t) + P_d(t) = -4 \qquad \text{for every } t \in G \setminus \{0\}. \tag{1}$$

Then $s_a, s_b, s_c, s_d \in \{\pm 1\}$. Write A, B, C, D for $[a], [b], [c], [d]$ and a, b, c, d also for the four sign sums, and put

$$M = \begin{pmatrix} A & BR & CR & DR \\ -BR & A & D^{\mathsf{T}}R & -C^{\mathsf{T}}R \\ -CR & -D^{\mathsf{T}}R & A & B^{\mathsf{T}}R \\ -DR & C^{\mathsf{T}}R & -B^{\mathsf{T}}R & A \end{pmatrix}, \qquad S = \begin{pmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{pmatrix},$$

$$H = \begin{pmatrix} -S & S \otimes \mathbf{1}^{\mathsf{T}} \\ S \otimes \mathbf{1} & M \end{pmatrix}.$$

Then H is a Hadamard matrix of order $4(m+1)$.

The matrix M is the Goethals–Seidel array [3] of the four group-developed matrices.

Proof. Each $P_x(0)$ equals m, so (1) gives $\sum_{t \in G}\big(P_a + P_b + P_c + P_d\big)(t) = 4m - 4(m-1) = 4$. Since $\sum_t P_x(t) = s_x^2$, we get $s_a^2 + s_b^2 + s_c^2 + s_d^2 = 4$. Each s_x is odd because m is odd, so each s_x is ± 1. In particular every entry of H is ± 1, and H has order $4 + 4m$.

By Lemma 2(2) and (1),

$$AA^{\mathsf{T}} + BB^{\mathsf{T}} + CC^{\mathsf{T}} + DD^{\mathsf{T}} = 4(m+1)I - 4J. \tag{2}$$

The array M. We show $MM^{\mathsf{T}} = I_4 \otimes \big(4(m+1)I - 4J\big)$. By Lemma 2(3), $(XR)^{\mathsf{T}} = XR$ and $(X^{\mathsf{T}}R)^{\mathsf{T}} = RX = X^{\mathsf{T}}R$ for $X \in \{A, B, C, D\}$, and $XRYR = XY^{\mathsf{T}}$ for all such X, Y, as $RYR = Y^{\mathsf{T}}$; similarly $X^{\mathsf{T}}RY^{\mathsf{T}}R = X^{\mathsf{T}}Y$ and $XRY^{\mathsf{T}}R = XY$. A diagonal block of MM^{T} is the sum of the four terms UU^{T} over the blocks U of one block row, and by these identities and Lemma 2(2) each is one of $AA^{\mathsf{T}}, BB^{\mathsf{T}}, CC^{\mathsf{T}}, DD^{\mathsf{T}}$, each appearing once. So every diagonal block equals (2). For the off-diagonal blocks we compute the $(1,2)$ block; the other five are the same computation with the letters permuted:

$$\begin{aligned} &A(-BR)^{\mathsf{T}} + BR\,A^{\mathsf{T}} + CR\,(D^{\mathsf{T}}R)^{\mathsf{T}} + DR\,(-C^{\mathsf{T}}R)^{\mathsf{T}} \\ &\quad = -ABR + BRA^{\mathsf{T}} + CR\,D^{\mathsf{T}}R - DR\,C^{\mathsf{T}}R. \end{aligned}$$

Here $BRA^{\mathsf{T}} = B(RA^{\mathsf{T}}R)R = BAR$, $CR\,D^{\mathsf{T}}R = CD$ and $DR\,C^{\mathsf{T}}R = DC$, so the block is $(BA - AB)R + CD - DC = 0$ by Lemma 2(1).

The border. The matrix S satisfies $SS^{\mathsf{T}} = (a^2 + b^2 + c^2 + d^2)I_4 = 4I_4$, by a direct check of its rows. Let $E = I_4 \otimes \mathbf{1}$, the $4m \times 4$ matrix whose jth column is the indicator of the jth block of columns. By Lemma 2(4), every block

of M has constant row sums, equal to the corresponding entry of S; for example $D^{\mathsf{T}}R\mathbf{1} = D^{\mathsf{T}}\mathbf{1} = d\mathbf{1}$. Hence $ME = ES$, that is, $M(I_4 \otimes \mathbf{1}) = S \otimes \mathbf{1}$.

The Gram matrix of H. Write $H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$ as displayed. Then

$$\begin{aligned} H_{11}H_{11}^{\mathsf{T}} + H_{12}H_{12}^{\mathsf{T}} &= SS^{\mathsf{T}} + (SS^{\mathsf{T}}) \otimes (\mathbf{1}^{\mathsf{T}}\mathbf{1}) = 4(m+1)I_4, \\ H_{11}H_{21}^{\mathsf{T}} + H_{12}H_{22}^{\mathsf{T}} &= -(SS^{\mathsf{T}}) \otimes \mathbf{1}^{\mathsf{T}} + (ME\,S^{\mathsf{T}})^{\mathsf{T}} = -4E^{\mathsf{T}} + 4E^{\mathsf{T}} = 0, \\ H_{21}H_{21}^{\mathsf{T}} + H_{22}H_{22}^{\mathsf{T}} &= (SS^{\mathsf{T}}) \otimes J + MM^{\mathsf{T}} = 4(m+1)I_{4m}. \end{aligned}$$

In the second line we used $H_{12}H_{22}^{\mathsf{T}} = (S \otimes \mathbf{1}^{\mathsf{T}})M^{\mathsf{T}} = (M(S^{\mathsf{T}} \otimes \mathbf{1}))^{\mathsf{T}}$, $S^{\mathsf{T}} \otimes \mathbf{1} = ES^{\mathsf{T}}$ and $ME\,S^{\mathsf{T}} = ESS^{\mathsf{T}} = 4E$. In the third line, $(SS^{\mathsf{T}}) \otimes J = 4\,I_4 \otimes J$ and $MM^{\mathsf{T}} = I_4 \otimes \big(4(m+1)I - 4J\big)$. So $HH^{\mathsf{T}} = 4(m+1)I$. □

The condition (1) has two standard reformulations. Let $D_x = \{g : x(g) = -1\}$ be the negative set of x and $\lambda_x(t) = |\{(g,h) \in D_x^2 : h - g = t\}|$. Since $x = 1 - 2 \cdot 1_{D_x}$,

$$P_x(t) = m - 4|D_x| + 4\lambda_x(t). \tag{3}$$

So (1) says that the four negative sets form supplementary difference sets: every nonzero t arises the same number $\lambda = |D_a| + |D_b| + |D_c| + |D_d| - m - 1$ of times as a difference of two elements of one of the sets. Equivalently, by Fourier analysis on G, $|\hat{a}(\chi)|^2 + |\hat{b}(\chi)|^2 + |\hat{c}(\chi)|^2 + |\hat{d}(\chi)|^2 = 4(m+1)$ for every nontrivial character χ of G, where $\hat{x}(\chi) = \sum_g x(g)\chi(g)$. The search in Section 5 uses the second form on quotients of G.

3 The four functions

Let $K = \mathbb{Z}_7 \times \mathbb{Z}_{19}$ and let $\mathbb{F}_9 = \mathbb{F}_3[i]$ with $i^2 = -1$. We identify $\mathbb{F}_9$ with $\mathbb{F}_3^2$ by $u + vi \mapsto (u,v)$, and write the elements of

$$G = K \times \mathbb{F}_9 = \mathbb{Z}_7 \times \mathbb{Z}_{19} \times \mathbb{F}_3^2$$

as (x,y,u,v) with $x \in \mathbb{Z}_7$, $y \in \mathbb{Z}_{19}$ and $u,v \in \mathbb{F}_3 = \{0,1,2\}$. The group has order $m = 7 \cdot 19 \cdot 9 = 1197$, and $4(m+1) = 4792$.

Multipliers. The map $\mu(x,y) = (2x, 4y)$ is an automorphism of K of order 9: the element 2 has order 3 in $\mathbb{Z}_7^\times$ and 4 has order 9 in $\mathbb{Z}_{19}^\times$. Multiplication by i, which is $(u,v) \mapsto (-v,u)$, is an automorphism of $\mathbb{F}_9$ of order 4. The group $\langle\mu\rangle \times \langle i\rangle$ of order 36 acts on G coordinatewise. Its orbits on K are $\{(0,0)\}$, two orbits of size 3 in $\mathbb{Z}_7^\times \times \{0\}$, two orbits of size 9 in $\{0\} \times \mathbb{Z}_{19}^\times$,

and 12 orbits of size 9 in $\mathbb{Z}_7^\times \times \mathbb{Z}_{19}^\times$: 17 orbits in all. Order them by their least element in the lexicographic order. Their least elements are

j	0	1	2	3	4	5	6	7	8
(x,y)	$(0,0)$	$(0,1)$	$(0,2)$	$(1,0)$	$(1,1)$	$(1,2)$	$(1,4)$	$(1,5)$	$(1,8)$
j	9	10	11	12	13	14	15	16	
(x,y)	$(1,10)$	$(3,0)$	$(3,1)$	$(3,2)$	$(3,4)$	$(3,5)$	$(3,8)$	$(3,10)$	

and we write O_j for the orbit with index j. The orbits of $\langle i \rangle$ on $\mathbb{F}_9$ are $F_0 = \{0\}$, the set $F_1 = \{\pm 1, \pm i\}$ of nonzero squares, which are the points with exactly one nonzero coordinate, and the set $F_2 = \{\pm 1 \pm i\}$ of nonsquares, which are the points with two nonzero coordinates. So the 51 orbits of the multiplier group on G are the products $O_j \times F_k$.

The functions. Each of the four functions a, b, c, d is constant on these 51 orbits. For $x \in \{a, b, c, d\}$ and $k \in \{0, 1, 2\}$ let $w_{x,k}$ be the integer in Table 1. The value of x on $O_j \times F_k$ is -1 if bit j of $w_{x,k}$ is set, counting from the least significant bit as bit 0, and $+1$ otherwise.

Function	F_0 (origin)	F_1 (squares)	F_2 (nonsquares)	$\lvert D_x \rvert$	s_x
a	`0c85b`	`1c32a`	`03cdc`	598	1
b	`08573`	`07c3c`	`187c2`	598	1
c	`08328`	`09e3d`	`16d4b`	599	-1
d	`04ad2`	`03e65`	`1991b`	599	-1

Table 1: The twelve masks $w_{x,k}$, in hexadecimal, with the size of each negative set and each sign sum. Bit j of $w_{x,k}$ gives the sign of x on $O_j \times F_k$.

For example, the mask $w_{a,0} =$ `0c85b` has bits $0, 1, 3, 4, 6, 11, 14$ and 15 set. So $a(x, y, 0, 0) = -1$ exactly when (x, y) lies in one of $O_0, O_1, O_3, O_4, O_6, O_{11}, O_{14}, O_{15}$.

Lemma 4. *The functions a, b, c, d of Table 1 satisfy (1) on G: with the notation of (3), $\lambda_a(t) + \lambda_b(t) + \lambda_c(t) + \lambda_d(t) = 1196$ for every nonzero $t \in G$.*

Proof. This is a finite computation. The four negative sets have sizes $598, 598, 599, 599$, in total 2394. A program expands the masks, enumerates all $598^2 + 598^2 + 599^2 + 599^2 = 1\,432\,810$ ordered pairs (g, h) in the four negative sets, and tallies $h - g$. It finds 2394 pairs with $h - g = 0$ and exactly 1196 for each of the 1196 nonzero elements. By (3), $\sum_x P_x(t) = 4 \cdot 1197 - 4 \cdot 2394 + 4 \cdot 1196 = -4$ for $t \neq 0$. □

The multiplier symmetry reduces the check. If σ is an automorphism of G and $x \circ \sigma = x$, then $P_x(\sigma t) = \sum_g x(\sigma g)x(\sigma g + \sigma t) = P_x(t)$. So the sum in (1) is constant on the 50 nonzero orbits $O_j \times F_k$, and it suffices to check one shift in each. The programs check all 1196 shifts.

Proof of Theorem 1. Apply Theorem 3 to the group G of order 1197 and the functions of Table 1, which satisfy (1) by Lemma 4. The result is a Hadamard matrix of order $4 \cdot 1198 = 4792$. □

4 Verification

The four sign strings, the full matrix H and the programs used below are available at `https://github.com/charlieoneill11/maths-releases`. The matrix is stored as text, one row of H per line, with the entries 1 and -1 separated by commas and each line ending with a line feed. The rows and columns are in the order of Theorem 3: the four border rows, then the $4 \cdot 1197$ rows of $(S \otimes \mathbf{1} \;\; M)$, with G in lexicographic order of (x, y, u, v). This text has SHA-256 hash

`5953fc1f8ad4cfcd38c782b9e7be909af4b0d7dc5bd94073a98dc4765e57eab6`.

We ran four checks. The first proves Lemma 4. The other three check the matrix directly, so they do not rely on Theorem 3.

1. *Differences.* A program that uses only the Python standard library checks that the twelve masks expand to the four sign strings, and counts the differences as in the proof of Lemma 4.

2. *Row pairs.* A second program builds every entry of H from an explicit formula, packs each row into an integer, and checks all $\binom{4792}{2} = 11\,479\,236$ pairs of distinct rows by exclusive-or and population count. Every pair of rows differs in exactly 2396 positions, so every two distinct rows are orthogonal. It also writes the matrix it builds in the text form above, and the SHA-256 hash agrees.

3. *The array.* A third, separately written program builds A, B, C, D, M, S and H from the masks by the matrix formulas of Theorem 3 and computes HH^{T}. It also reads the stored matrix and computes its Gram matrix. Both Gram matrices equal $4792I$, and the matrix built from the masks equals the stored matrix entry by entry. The products are computed in double precision; every partial sum is an integer of absolute value at most 4792, so the arithmetic is exact.

4. *Kumar's checker.* Kumar's standalone verifier [6], written in C, reads the stored matrix, checks that every entry is ± 1, and computes all $11\,484\,028$ inner products of pairs of rows, the diagonal included. It reports no violations. As a control, changing a single entry makes it report 4791 violations.

Each check takes a few seconds.

5 How the functions were found

This section records the search. It is not needed for the proof.

For orders $8p$, Kumar [6] studied Spence-type structures over groups of odd order $2p-1$, with the four functions constant on the orbits of a group of multipliers, and estimated their cost. For $p = 599$ his best remaining structure was the group $G = \mathbb{Z}_7 \times \mathbb{Z}_{19} \times \mathbb{F}_9$ with a multiplier group of order 36, at an estimated $1.5 \cdot 10^3$ core-hours. The same approach gave his matrix of order 5912 over $\mathbb{Z}_{211} \times \mathbb{F}_7$. Before searching at order 4792 we reran his order-5912 search from the recorded seed and recovered his matrix.

A function invariant under the multipliers is determined by its three values on each fibre $\{k\} \times \mathbb{F}_9$, at 0, on the squares and on the nonsquares. Following Kumar, the search first fixed some coarse numerical data of each of the four functions, such as sums of its values over parts of the fibres. Up to the symmetries of the problem there are 1141 possibilities for these data, which we call cases. Projecting (1) to the quotients $\mathbb{Z}_7 \times \mathbb{F}_9$ and $\mathbb{Z}_{19} \times \mathbb{F}_9$ of G, in the Fourier form at the end of Section 2, gives exact necessary conditions, and 524 of the 1141 cases satisfy them. For a surviving case the search enumerated candidate functions block by block, matched the four blocks on their projected autocorrelations, and tested each match against (1) on all of G.

The search ran on local CPUs with two choices of multiplier on K, $(2,4)$ and $(2,5)$, which give the same projected conditions. In all, 250 cases for $(2,4)$ and 80 for $(2,5)$ ended without a solution, and two cases for $(2,4)$ gave solutions. Case 268 gave the functions of Table 1. A clean rebuild and rerun of that case gave the same four sign strings. Case 523 for $(2,4)$ gave a second solution, whose negative sets all have size 599. It also passes the first two checks of Section 4. We have not tested whether the two Hadamard matrices are equivalent under signed permutations of rows and columns. The second certificate and the search programs are also available at `https://github.com/charlieoneill11/maths-releases`.

6 Remarks

Order $2396 = 4 \cdot 599$, the smallest open order, is still open. Kumar [6] discusses which structures remain for it. Of the orders $8p$ below $10\,000$ in Kumar's list, only $5752 = 8 \cdot 719$ is now open. For $p = 719$ the corresponding group has order 1437, and Kumar reports that several structures of this type are already excluded there.

Acknowledgements

This work builds directly on Kishan Kumar's public search programs, search records and verifier [6], and the structure used here was identified in his search records. We are grateful to him for making them public.

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